STATISTICS IN TRANSITION new series, March 2018
Vol. 19, No. 1, pp. 135–148, DOI 10.21307/stattrans-2018-008

POWER ISHITA DISTRIBUTION AND ITS APPLICATION TO MODEL LIFETIME DATA

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ABSTRACT

A study on two-parameter power Ishita distribution (PID), of which Ishita distribution introduced by Shanker and Shukla (2017 a) is a special case, has been carried out and its important statistical properties including shapes of the density, moments, skewness and kurtosis measures, hazard rate function, and stochastic ordering have been discussed. The maximum likelihood estimation has been discussed for estimating its parameters. An application of the distribution has been explained with a real lifetime data from engineering, and its goodness of fit shows better fit over two-parameter power Akash distribution (PAD), two-parameter power Lindley distribution (PLD) and one-parameter Ishita, Akash, Lindley and exponential distributions.

Key words: Ishita distribution, moments, hazard rate function, stochastic ordering, maximum likelihood estimation, goodness of fit.

1. Introduction

The probability density function (pdf) of Ishita distribution introduced by Shanker and Shukla (2017 a) is given by

$$f_1(y;\theta)=\frac{\theta^3}{\theta^3+2}\left(\theta+y^2\right)e^{-\theta y} \quad ;y>0,\ \theta>0 \tag{1.1}$$

$$=p\,g_1(y;\theta)+(1-p)\,g_2(y;\theta) \tag{1.2}$$

where $$p=\frac{\theta^3}{\theta^3+2}$$

$$g_1(y;\theta)=\theta e^{-\theta y} \quad ;y>0,\theta>0$$

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$$g_2\left(y;\theta\right)=\frac{\theta^3}{\Gamma(3)}e^{-\theta y}y^{3-1}\;;y>0,\theta>0$$

The pdf in (1.1) reveals that the Ishita distribution is a two-component mixture of an exponential distribution (with scale parameter θ) and a gamma distribution (with shape parameter 2 and scale parameter θ), with mixing proportion $p=\frac{\theta^3}{\theta^3+2}$. Shanker and Shukla (2017 a) have discussed some of its mathematical and statistical properties including its shapes for varying values of the parameter, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic ordering, mean deviations, order statistics, Bonferroni and Lorenz curves, Renyi entropy measure, stress-strength reliability, and the applications of the distribution for modelling lifetime data from engineering and medical science. However, there are some situations where the Ishita distribution may not be suitable from either theoretical or applied point of view. Shukla and Shanker (2017) have also obtained a Poisson mixture of Ishita distribution and named it Poisson-Ishita distribution, and studied its various statistical properties, estimation of parameter and the goodness of fit with some real count data sets.

The corresponding cumulative distribution function (cdf) of (1.1) is given by

$$F_1\left(y;\theta\right)=1-\left[1+\frac{\theta y\left(\theta y+2\right)}{\theta^3+2}\right]e^{-\theta y};y>0,\theta>0 \tag{1.3}$$

Recall that the pdf and the cdf of two-parameter power Akash distribution (PAD) introduced by Shanker and Shukla (2017 b) and two-parameter power Lindley distribution (PLD) introduced by Ghitany *et al.* (2013) are respectively given by

$$f_2\left(x;\theta,\alpha\right)=\frac{\alpha\,\theta^3}{\theta^2+2}\left(1+x^{2\alpha}\right)x^{\alpha-1}e^{-\theta x^\alpha}\;;x>0,\theta>0,\alpha>0 \tag{1.4}$$

$$F_2\left(x;\theta,\alpha\right)=1-\left[1+\frac{\theta x^\alpha\left(\theta x^\alpha+2\right)}{\theta^2+2}\right]e^{-\theta x^\alpha}\;;x>0,\theta>0,\alpha>0 \tag{1.5}$$

$$f_3\left(x;\theta,\alpha\right)=\frac{\alpha\,\theta^2}{\theta+1}\left(1+x^{\alpha}\right)x^{\alpha-1}e^{-\theta x^\alpha}\;;x>0,\theta>0,\alpha>0 \tag{1.6}$$

$$F_3\left(x;\theta,\alpha\right)=1-\left[1+\frac{\theta x^\alpha}{\theta+1}\right]e^{-\theta x^\alpha}\;;x>0,\theta>0,\alpha>0 \tag{1.7}$$

A detailed study regarding various properties, estimation of parameters and applications of PAD and PLD can be seen from Shanker and Shukla (2017 b) and

Ghitany *et al.* (2013) respectively. At $\alpha = 1$, PAD reduces to Akash distribution introduced by Shanker (2015) having pdf and cdf given by

$$f_4(x;\theta) = \frac{\theta^3}{\theta^2 + 2}\left(1 + x^2\right)e^{-\theta x} \; ; x > 0, \theta > 0 \tag{1.8}$$

$$F_4(x;\theta) = 1 - \left[1 + \frac{\theta x(\theta x + 2)}{\theta^2 + 2}\right]e^{-\theta x} ; x > 0, \theta > 0 \tag{1.9}$$

Shanker (2015) has a detailed study about various statistical and mathematical properties of Akash distribution, estimation of parameter and applications for modelling lifetime data from engineering and medical science and showed that Akash distribution gives better fit than both exponential and Lindley distributions. Shanker (2017) has also obtained a Poisson mixture of Akash distribution and named Poisson-Akash distribution and discussed important statistical properties, estimation of parameter using both the method of moments and the method of maximum likelihood and the application for modelling count data.

Similarly, at $\alpha = 1$, PLD reduces to Lindley distribution introduced by Lindley (1958) having pdf and cdf given by

$$f_5(x;\theta) = \frac{\theta^2}{\theta + 1}(1 + x)e^{-\theta x}; x > 0, \theta > 0 \tag{1.10}$$

$$F_5(x;\theta) = 1 - \left[1 + \frac{\theta x}{\theta + 1}\right]e^{-\theta x}; x > 0, \theta > 0 \tag{1.11}$$

Ghitany *et al*. (2008) have a detailed study about various properties of Lindley distribution, estimation of parameter and application for modelling waiting time data from a bank and it has been shown that it gives better fit than exponential distribution. Shanker *et al.* (2016) have a detailed and critical comparative study of modelling real lifetime data from engineering and biomedical sciences using Akash, Lindley and exponential distribution and observed that each of these one-parameter distribution has some advantage over the other but none is perfect for modelling all real lifetime data. Since Ishita distribution gives better fit than Akash, Lindley and exponential distribution, it is expected and hoped that the two-parameter power Ishita distribution (PID) will provide a better model over two-parameter power Akash distribution (PAD) and power Lindley distribution (PLD) and one-parameter Ishita, Akash, Lindley and exponential distributions.

In this paper, a two-parameter power Ishita distribution (PID), which includes one-parameter Ishita distribution, has been introduced and its various properties including shapes for varying values of the parameters, survival function, hazard rate function, moments, stochastic ordering have been studied. The maximum likelihood estimation has been discussed for estimating its parameters. Finally, applications and goodness of fit of PID has been illustrated with a real life time data and fit has been found better over two-parameter power Akash distribution

(PAD) of Shanker and Shukla (2017 b), two-parameter power Lindley distribution (PLD) of Ghitany *et al.* (2013), and one-parameter Ishita, Akash, Lindley and exponential distributions.

2. Power Ishita distribution

Taking the power transformation $X = Y^{1/\alpha}$ in (1.1), pdf of the random variable X can be obtained as

$$f_6(x;\theta,\alpha) = \frac{\alpha\,\theta^3}{\theta^3+2}\left(\theta + x^{2\alpha}\right)x^{\alpha-1}\,e^{-\theta x^{\alpha}} \;;x>0,\theta>0,\alpha>0 \tag{2.1}$$

$$= p\,g_3(x;\theta,\alpha) + (1-p)\,g_4(x;\theta,\alpha) \tag{2.2}$$

where $$p = \frac{\theta^3}{\theta^3+2}$$

$$g_3(x;\theta,\alpha) = \alpha\,\theta\,x^{\alpha-1}e^{-\theta x^{\alpha}} \;;x>0,\alpha>0,\theta>0$$

$$g_4(x;\theta,\alpha) = \frac{\alpha\,\theta^3 x^{3\alpha-1}e^{-\theta x^{\alpha}}}{2};x>0,\alpha>0,\theta>0$$

We would call the density in (2.1) “Power Ishita distribution (PID)” and denote it as PID(θ,α). It is obvious that the PID is also a two-component mixture of Weibull distribution (with shape parameter α and scale parameter θ), and a generalized gamma distribution (with shape parameters 3, α and scale parameter θ) introduced by Stacy (1962) with their mixing proportion $p = \frac{\theta^3}{\theta^3+2}$.
The corresponding cumulative distribution function (cdf) of (2.1) can be obtained as

$$F_6(x;\theta,\alpha) = 1 - \left[1 + \frac{\theta x^{\alpha}\left(\theta x^{\alpha}+2\right)}{\theta^3+2}\right]e^{-\theta x^{\alpha}} \;;x>0,\theta>0,\alpha>0 \tag{2.3}$$

Graphs of the pdf and the cdf of PID for varying values of the parameters have been drawn and presented in Figures 1 and 2 respectively. If $\alpha = 1$, the pdf of PID is monotonically decreasing for increasing values of the parameter θ. But for $\alpha > 1$ and increasing values of the parameter θ, the shapes of the pdf of PID become negatively skewed, positively skewed, symmetrical, platykurtic and mesokurtic; and this means that PID can be used for modelling lifetime data of various nature.

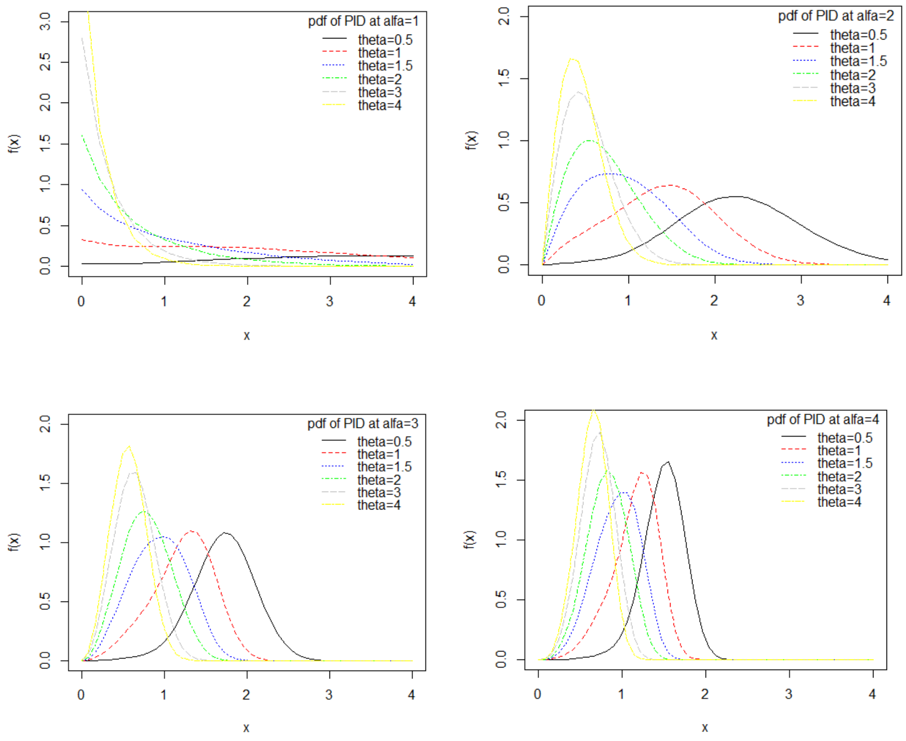
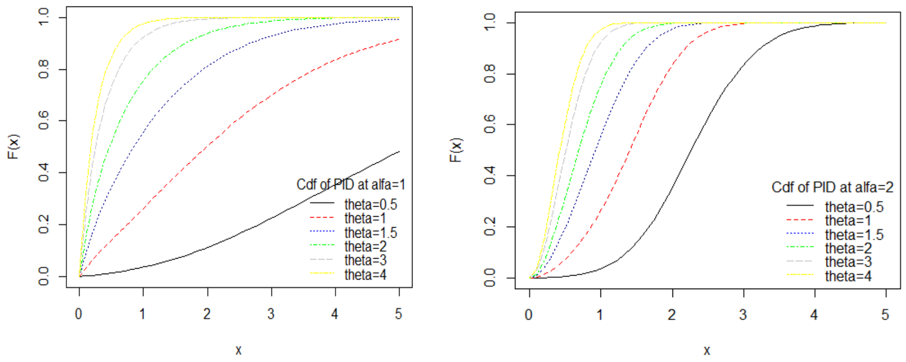


Figure.1. Graphs of pdf of PID for varying values of parameters θ and α



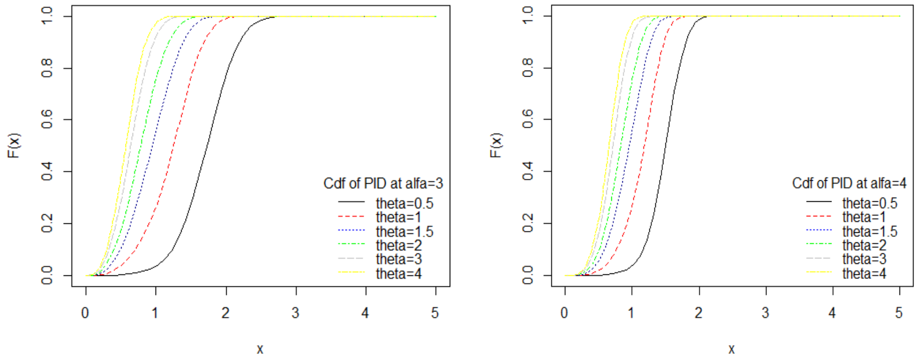


Figure 2. Graphs of cdf of PID for varying values of parameters θ and α

3. Survival and hazard rate functions

The survival function, $S(x)$ and hazard rate function, $h(x)$ of the PID can be obtained as

$$S(x;\theta,\alpha)=1-F_6(x;\theta,\alpha)=\left[\frac{\theta x^{\alpha}\left(\theta x^{\alpha}+2\right)+\left(\theta^3+2\right)}{\theta^3+2}\right]e^{-\theta x^{\alpha}}\ ;x>0,\theta>0,\alpha>0 \tag{3.1}$$

$$h(x;\theta,\alpha)=\frac{f_6(x;\theta,\alpha)}{S(x;\theta,\alpha)}=\frac{\alpha\theta^3\left(1+x^{2\alpha}\right)x^{\alpha-1}}{\theta x^{\alpha}\left(\theta x^{\alpha}+2\right)+\left(\theta^3+2\right)};\quad x>0,\ \theta>0,\ \alpha>0 \tag{3.2}$$

The nature and behaviour of $h(x)$ of the PID for varying values of the parameters θ and α are shown graphically in Figure 3. It is obvious from the graphs of $h(x)$ that it is monotonically decreasing and increasing for increased values of the parameters θ and α.

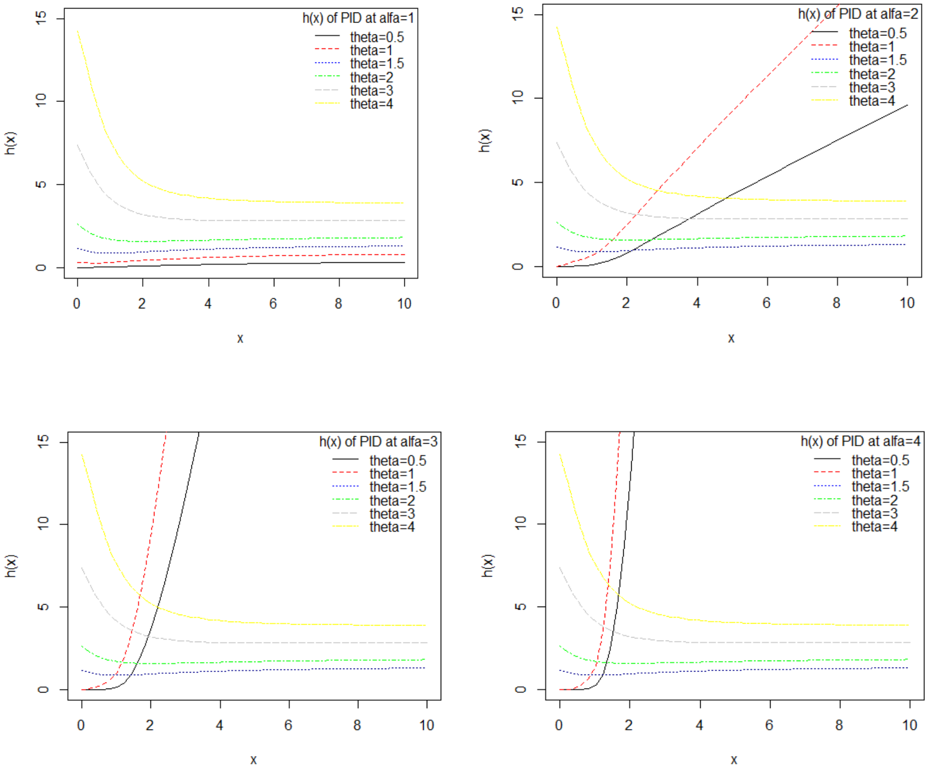


Figure 3. Graphs of $h(x)$ of PID for varying values of the parameters θ and α

4. Moments and related measures

Using the mixture representation (2.2), the r th moment about origin of the PID can be obtained as

$$\mu_r' = E\left(X^r\right) = p\int_0^\infty x^r g_3\left(x;\theta,\alpha\right)dx + \left(1-p\right)\int_0^\infty x^r g_4\left(x;\theta,\alpha\right)dx$$

$$= \frac{r\Gamma\left(\dfrac{r}{\alpha}\right)\left[\alpha^2\theta^3 + \left(r+\alpha\right)\left(r+2\alpha\right)\right]}{\alpha^3\theta^{r/\alpha}\left(\theta^3+2\right)}; r = 1, 2, 3, ... \tag{4.1}$$

It should be noted that at $\alpha = 1$, the above expression will reduce to the r th moment about origin of Ishita distribution and is given by

$$\mu_r' = \frac{r!\left[\theta^3 + (r+1)(r+2)\right]}{\theta^r\left(\theta^3+2\right)};r=1,2,3,...$$

Therefore, the mean and the variance of the PID are obtained as

$$\mu_1' = \frac{\Gamma\left(\frac{1}{\alpha}\right)\left[\alpha^2\theta^3 + (\alpha+1)(2\alpha+1)\right]}{\alpha^3\theta^{1/\alpha}\left(\theta^3+2\right)}$$

$$\sigma^2 = \frac{2\Gamma\left(\frac{2}{\alpha}\right)\left[\alpha^2\theta^3 + 2(\alpha+1)(\alpha+2)\right]\alpha^3\left(\theta^3+2\right) - \left(\Gamma\left(\frac{1}{\alpha}\right)\right)^2\left[\alpha^2\theta^3 + (\alpha+1)(2\alpha+1)\right]^2}{\alpha^6\theta^{2/\alpha}\left(\theta^3+2\right)^2}$$

The coefficient of skewness and the coefficient of kurtosis of PID, upon substituting for the raw moments and standard deviation (σ), can be obtained using following expressions

$$\text{Coefficient of Skewness} = \frac{\mu_3' - 3\mu_2'\mu_1' + 2\left(\mu_1'\right)^3}{\sigma^3}$$

and Coefficient of Kurtosis $= \dfrac{\mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\left(\mu_1'\right)^2 - 3\left(\mu_1'\right)^4}{\sigma^4}$.

5. Stochastic ordering

Stochastic ordering of positive continuous random variables is an important tool for judging their comparative behaviour. A random variable X is said to be smaller than a random variable Y in the

(i) stochastic order $\left(X \leq_{st} Y\right)$ if $F_X(x) \geq F_Y(x)$ for all x

(ii) hazard rate order $\left(X \leq_{hr} Y\right)$ if $h_X(x) \geq h_Y(x)$ for all x

(iii) mean residual life order $\left(X \leq_{mrl} Y\right)$ if $m_X(x) \leq m_Y(x)$ for all x

(iv) likelihood ratio order $\left(X \leq_{lr} Y\right)$ if $\dfrac{f_X(x)}{f_Y(x)}$ decreases in x.

The following important interrelationships due to Shaked and Shanthikumar (1994) are well known for establishing stochastic ordering of distributions

$$X \leq_{lr} Y \Rightarrow X \leq_{hr} Y \Rightarrow X \leq_{mrl} Y$$
$$\Downarrow$$
$$X \leq_{st} Y$$

The PID is ordered with respect to the strongest 'likelihood ratio ordering' as shown in the following theorem:

Theorem: Let X ~ PID$\left(\theta_1,\alpha_1\right)$ and Y ~ PID$\left(\theta_2,\alpha_2\right)$. If $\theta_1 > \theta_2$ and $\alpha_1 = \alpha_2$ (or $\alpha_1 < \alpha_2$ and $\theta_1 = \theta_2$) then $X \leq_{lr} Y$ and hence $X \leq_{hr} Y$, $X \leq_{mrl} Y$ and $X \leq_{st} Y$.

Proof: From the pdf of PID (2.1), we have

$$\frac{f_X\left(x,\theta_1,\alpha_1\right)}{f_Y\left(x;\theta_2,\alpha_2\right)} = \left(\frac{\alpha_1\theta_1^3\left(\theta_2^3+2\right)}{\alpha_2\theta_2^3\left(\theta_1^3+2\right)}\right)\left(\frac{\theta_1+x^{2\alpha_1}}{\theta_2+x^{2\alpha_2}}\right)x^{\alpha_1-\alpha_2}e^{-\left(\theta_1 x^{\alpha_1}-\theta_2 x^{\alpha_2}\right)} \quad ; \; x>0$$

Now

$$\ln\frac{f_X\left(x,\theta_1,\alpha_1\right)}{f_Y\left(x;\theta_2,\alpha_2\right)} = \ln\left(\frac{\alpha_1\theta_1^3\left(\theta_2^3+2\right)}{\alpha_2\theta_2^3\left(\theta_1^3+2\right)}\right)+\ln\left(\frac{\theta_1+x^{2\alpha_1}}{\theta_2+x^{2\alpha_2}}\right)+\left(\alpha_1-\alpha_2\right)\ln x-\left(\theta_1 x^{\alpha_1}-\theta_2 x^{\alpha_2}\right)$$

This gives

$$\frac{d}{dx}\left\{\ln\frac{f_X\left(x,\theta_1,\alpha_1\right)}{f_Y\left(x;\theta_2,\alpha_2\right)}\right\} = \frac{2\left(\alpha_1\theta_2 x^{2\alpha_1-1}-\alpha_2\theta_1 x^{2\alpha_2-1}\right)+2\left(\alpha_1-\alpha_2\right)x^{2\left(\alpha_1+\alpha_2\right)-1}}{\left(\theta_1+x^{2\alpha_1}\right)\left(\theta_2+x^{2\alpha_2}\right)}+\frac{\alpha_1-\alpha_2}{x}$$
$$-\left(\alpha_1\theta_1 x^{\alpha_1-1}-\alpha_2\theta_2 x^{\alpha_2-1}\right)$$

Clearly for $\theta_1 > \theta_2$ and $\alpha_1 = \alpha_2$ (or $\alpha_1 < \alpha_2$ and $\theta_1 = \theta_2$), $\frac{d}{dx}\left\{\ln\frac{f_X\left(x,\theta_1,\alpha_1\right)}{f_Y\left(x;\theta_2,\alpha_2\right)}\right\} < 0$. This means that $X \leq_{lr} Y$ and hence $X \leq_{hr} Y$, $X \leq_{mrl} Y$ and $X \leq_{st} Y$. Thus PID follows the strongest likelihood ratio ordering.

6. Maximum likelihood estimation

Let $\left(x_1, x_2, x_3, \ldots, x_n\right)$ be a random sample of size n from PID$\left(\theta,\alpha\right)$. Then, the log-likelihood function is given by

$$\ln L = \sum_{i=1}^{n}\ln f_6\left(x_i;\theta,\alpha\right)$$

$$= n\left[\ln\alpha + 3\ln\theta - \ln\left(\theta^3 + 2\right)\right] + \sum_{i=1}^{n}\ln\left(\theta + x_i^{2\alpha}\right) + (\alpha - 1)\sum_{i=1}^{n}\ln(x_i) - \theta\sum_{i=1}^{n} x_i^{\alpha}\ .$$

The maximum likelihood estimates (MLE) $\left(\hat{\theta},\hat{\alpha}\right)$ of (θ,α) of PID (2.1) are the solutions of the following log likelihood equations

$$\frac{\partial \ln L}{\partial \theta} = \frac{3n}{\theta} - \frac{3n\theta^2}{\theta^3 + 2} + \sum_{i=1}^{n}\frac{1}{\left(\theta + x_i^{2\alpha}\right)} - \sum_{i=1}^{n} x_i^{\alpha} = 0$$

$$\frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha} + 2\sum_{x=1}^{n}\frac{x_i^{2\alpha}\ln(x_i)}{\left(\theta + x_i^{2\alpha}\right)} + \sum_{i=1}^{n}\ln(x_i) - \theta\sum_{i=1}^{n} x_i^{\alpha}\ln(x_i) = 0$$

These two log likelihood equations do not seem to be solved directly because these cannot be expressed in closed form. However, Fisher's scoring method can be applied to solve these equations iteratively. Thus, we have

$$\frac{\partial^2 \ln L}{\partial \theta^2} = -\frac{3n}{\theta^2} + \frac{3n\left(\theta^3 - 4\right)\theta}{\left(\theta^3 + 2\right)^2} - \sum_{i=1}^{n}\frac{1}{\left(\theta + x_i^{2\alpha}\right)^2}$$

$$\frac{\partial^2 \ln L}{\partial \theta\,\partial \alpha} = -2\sum_{i=1}^{n}\frac{x_i^{2\alpha}\ln(x_i)}{\left(\theta + x_i^{2\alpha}\right)^2} - \sum_{i=1}^{n} x_i^{\alpha}\ln(x_i) = \frac{\partial^2 \ln L}{\partial \alpha\,\partial \theta}$$

$$\frac{\partial^2 \ln L}{\partial \alpha^2} = -\frac{n}{\alpha^2} + 4\theta\sum_{i=1}^{n}\frac{\left(x_i\ln(x_i)\right)^2}{\left(\theta + x_i^{2\alpha}\right)^2} - \theta\sum_{i=1}^{n} x_i^{\alpha}\left(\ln(x_i)\right)^2$$

The MLE $\left(\hat{\theta},\hat{\alpha}\right)$ of (θ,α) of PID (2.1) are the solution of the following equations

$$\begin{bmatrix} \dfrac{\partial^2 \ln L}{\partial \theta^2} & \dfrac{\partial^2 \ln L}{\partial \theta\,\partial \alpha} \\ \dfrac{\partial^2 \ln L}{\partial \theta\,\partial \alpha} & \dfrac{\partial^2 \ln L}{\partial \alpha^2} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0\\ \hat{\alpha}=\alpha_0}} \begin{bmatrix} \hat{\theta} = \theta_0 \\ \hat{\alpha} = \alpha_0 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \ln L}{\partial \theta} \\ \dfrac{\partial \ln L}{\partial \alpha} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0\\ \hat{\alpha}=\alpha_0}}$$

where $\theta_0 \text{ and } \alpha_0$ are initial values of $\theta \text{ and } \alpha$. These equations are solved iteratively until sufficiently close estimates of $\hat{\theta} \text{ and } \hat{\alpha}$ are obtained. In this paper, R-software has been used to estimate the parameters $\theta \text{ and } \alpha$ for the considered dataset.

7. Applications and goodness of FIT

In this section, we present the goodness of fit of PID using maximum likelihood estimates of parameters to a real data set from engineering and compare its fit with the one-parameter exponential, Lindley, Akash and Ishita distributions and two-parameter PAD and PLD. The following real lifetime data have been considered for the goodness of fit of the considered distributions.

Data Set: The following data represent the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20mm, Bader and Priest (1982)

1.312 1.314 1.479 1.552 1.700 1.803 1.861 1.865 1.944 1.958 1.966 1.997
2.006 2.021 2.027 2.055 2.063 2.098 2.140 2.179 2.224 2.240 2.253 2.270
2.272 2.274 2.301 2.301 2.359 2.382 2.382 2.426 2.434 2.435 2.478 2.490
2.511 2.514 2.535 2.554 2.566 2.570 2.586 2.629 2.633 2.642 2.648 2.684
2.697 2.726 2.770 2.773 2.800 2.809 2.818 2.821 2.848 2.880 2.954 3.012
3.067 3.084 3.090 3.096 3.128 3.233 3.433 3.585 3.585

In order to compare the considered distributions, values of $-2\ln L$, AIC (Akaike Information Criterion), K-S Statistic (Kolmogorov-Smirnov Statistic) and p-value for the real dataset have been computed using maximum likelihood estimates and presented in Table 1. The formulae for computing AIC and K-S Statistics are as follows:
$AIC = -2\ln L + 2k$ and $\text{K-S} = \operatorname{Sup}_x \left| F_n(x) - F_0(x) \right|$, where k = the number of parameters, n = the sample size, $F_n(x)$ is the empirical (sample) cumulative distribution function and $F_0(x)$ is the theoretical cumulative distribution function. The best distribution is the distribution corresponding to lower values of $-2\ln L$, AIC, and K-S statistics and higher p-value.

Table 1. MLE's, -2ln L, AIC, K-S and p-value of the fitted distributions of the considered dataset

Model	ML Estimates	-2ln L	AIC	K-S	p-value
PID	$\hat{\theta} = 0.18063$ $\hat{\alpha} = 3.00429$	97.84	101.84	0.033	1.00
PAD	$\hat{\theta} = 0.169$ $\hat{\alpha} = 3.061$	98.02	102.02	0.038	0.999
PLD	$\hat{\theta} = 0.050$ $\hat{\alpha} = 3.868$	98.12	102.12	0.044	0.998
Ishita	$\hat{\theta} = 0.39152$	223.14	225.14	0.331	0.003

Table 1. MLE's, -2ln L, AIC, K-S and p-value of the fitted distributions of the considered dataset (cont.)

Model	ML Estimates	-2ln L	AIC	K-S	p-value
Akash	$\hat{\theta}=0.964726$	224.28	226.28	0.348	0.001
Lindley	$\hat{\theta}=0.659000$	238.38	240.38	0.390	0.000
Exponential	$\hat{\theta}=0.407941$	261.74	263.74	0.434	0.000

It is obvious from the goodness of fit based on K-S statistic that PID gives better fit than all the considered distributions and hence it can be considered an important two-parameter lifetime distribution for modelling lifetime data. The variance-covariance matrix and the 95% confidence intervals (CI's) of the ML estimates of the parameters θ and α of PID are presented in Table 2.

Table 2. Variance-Covariance matrix and 95% confidence intervals (CI's) for the parameters $\hat{\theta}$ and $\hat{\alpha}$ of the considered dataset

Parameters	Variance-Covariance Matrix		95% CI	
	$\hat{\theta}$	$\hat{\alpha}$	Lower	Upper
$\hat{\theta}$	0.002233	-0.0117269	0.105235	0.292116
$\hat{\alpha}$	-0.0117269	0.0662314	2.527340	3.53244

The profile of likelihood estimates of parameters $\hat{\theta}$ and $\hat{\alpha}$ of PID for the considered data set is shown in Figure 4. Also, the fitted plots of the considered dataset for PID are shown in Figure 5.

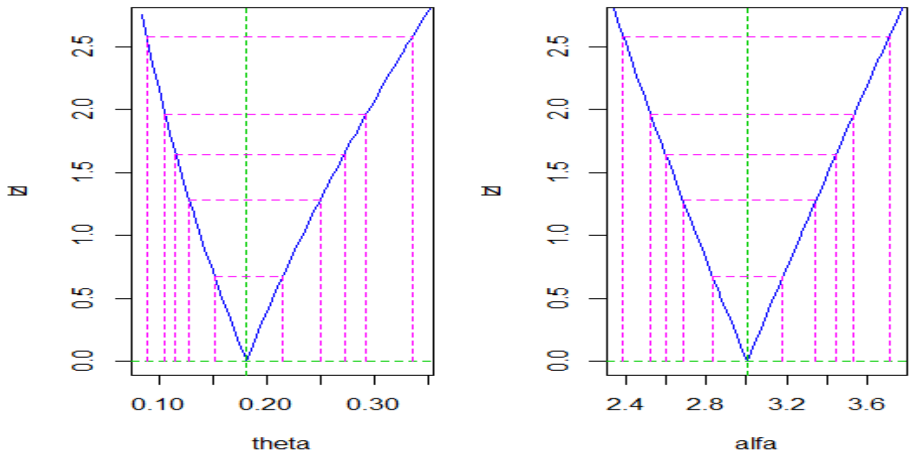


Figure 4. Profile of the likelihood estimates $\hat{\theta}$ and $\hat{\alpha}$ of PID for the considered dataset

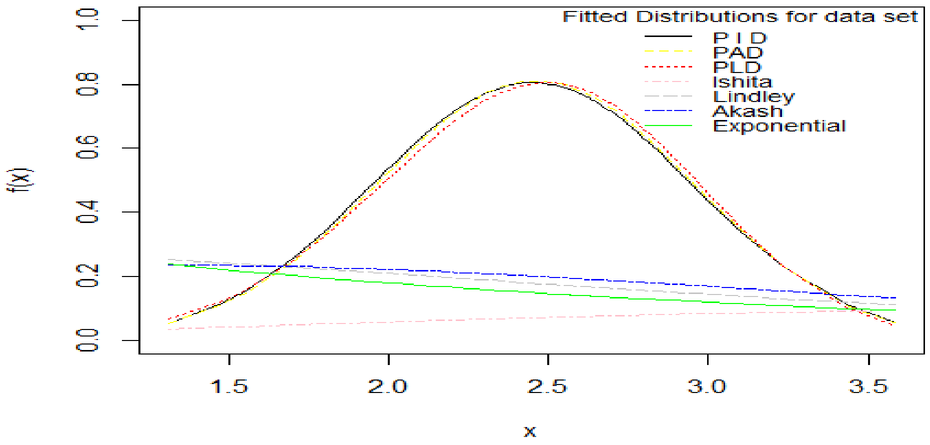


Figure 5. Fitted plots of the considered distributions for the given dataset

8. Concluding remarks

In this paper a two-parameter power Ishita distribution (PID), of which one-parameter Ishita distribution introduced by Shanker and Shukla (2017 a) is a special case, has been introduced and its important statistical properties including shapes of the density, moments, skewness and kurtosis measures and hazard rate function have been discussed. The stochastic ordering of the distribution has been studied. The maximum likelihood estimation has been discussed for estimating its parameters. An application and goodness of fit of PID have been discussed with a real lifetime data set from engineering and the fit has been found quite satisfactory over two-parameter PAD and PLD and one-parameter Ishita, Akash, Lindley and exponential distributions.

Acknowledgment

Authors are grateful to the Editor-In-Chief of the journal and two independent reviewers for their useful comments, which improved the presentation and the quality of the paper.

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